Exercise 24

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to a} c = c$$

Solution

According to Definition 2, proving this limit is logically equivalent to proving that

$$\text{if} \quad |x - a| < \delta \qquad \text{then} \qquad |c - c| < \varepsilon$$

for all positive ε. Assuming that $|x - a| < \delta$,

$$\begin{aligned} |c - c| &= |0| \\ &= 0 \\ &< \varepsilon. \end{aligned}$$

Therefore, by the precise definition of a limit,

$$\lim_{x \to a} c = c.$$